Hong Kong Mathematics Olympiad (2018/19)

Finals (Group – Event 1)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	Min. / Sec.
			Total score					

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

1. Let $x+y=32$ with $x, y \geq 0$. If a is the maximum value of $\sqrt{x}+\sqrt{y}$, determine the value of a.
已知 $x+y=32$，其中 $x, y \geq 0$。若 a 为 $\sqrt{x}+\sqrt{y}$ 的最大值，求 a 的值。

$a =$

2. A box contains only x one-dollar coins, $x+2$ two-dollar coins and $x+4$ five-dollar coins. If the probability of drawing a one-dollar coin randomly from the box is less than 0.1. If the box contains b coins, determine the value of b.
一个盒中只有 x 个一元硬币，$x+2$ 个二元硬币及 $x+4$ 个五元硬币。随机从盒中拿出一元硬币的概率小于 0.1。若盒中有 b 个硬币，求 b 的值。

$b =$

3. If c is the greatest common factor of the following numbers

$$3^3-3,\ 5^5-5,\ 7^7-7,\ 9^9-9,\ \ldots,\ 2019^{2019}-2019,$$

determine the value of c.
若 c 是以下数的最大公因子，

$$3^3-3,\ 5^5-5,\ 7^7-7,\ 9^9-9,\ \ldots,\ 2019^{2019}-2019,$$

求 c 的值。

$c =$

4. Let $x=\dfrac{\sqrt{5}+\sqrt{7}}{\sqrt{5}-\sqrt{7}}$ and $y=\dfrac{\sqrt{5}-\sqrt{7}}{\sqrt{5}+\sqrt{7}}$. If $d=3x^2-7xy+3y^2$, determine the value of d.

设 $x=\dfrac{\sqrt{5}+\sqrt{7}}{\sqrt{5}-\sqrt{7}}$ 和 $y=\dfrac{\sqrt{5}-\sqrt{7}}{\sqrt{5}+\sqrt{7}}$。若 $d=3x^2-7xy+3y^2$，求 d 的值。

$d =$

Hong Kong Mathematics Olympiad (2018/19)

Finals (Group – Event 2)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	Min.	Sec.
			Total score					

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

1. Let
$$X=\sqrt{2020-\sqrt{A}}$$
be a positive integer. Determine the least value of A.
设
$$X=\sqrt{2020-\sqrt{A}}$$
为正整数，求 A 的最小值。

$A=$

2. Suppose that
$$\begin{cases} x+y=5 \\ 4x^2+y^2=80 \end{cases}$$
and $P=(x_1,y_1)$ and $Q=(x_2,y_2)$ simultaneously satisfy these two equations. If $B=y_1-x_1+y_2-x_2$, determine the value of B.
假设
$$\begin{cases} x+y=5 \\ 4x^2+y^2=80 \end{cases}$$
及 $P=(x_1,y_1)$ 和 $Q=(x_2,y_2)$ 同时满足这两个等式。若 $B=y_1-x_1+y_2-x_2$，求 B 的值。

$B=$

3. If $Q=a^b-b^a$ is a positive integer, determine the least value of Q.

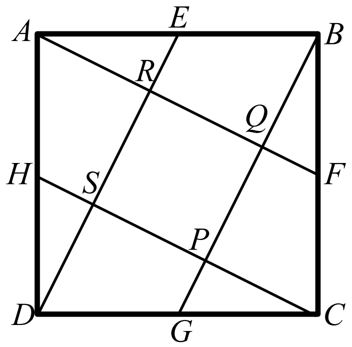
若 $Q=a^b-b^a$ 为正整数，求 Q 的最小值。

$n=$

4. In square $ABCD$, E, F, G and H are the mid-points of AB, BC, CD and AD respectively. DE intersects with AF and CH at R and S respectively. Moreover, BG intersects with AF and CH at Q and P respectively. If U is the area of square $ABCD$

and V is the area of quadrilateral $PQRS$, determine the value of $W = \dfrac{U}{V}$.

在正方形 $ABCD$ 中, E, F, G 和 H 分别是 AB, BC, CD 和 AD 的中点。DE 分别与线段 AF 和 CH 相交于点 R 和 S。BG 分别与线段 AF 和 CH 相交于点 Q 和 P。若 U 是正方形 $ABCD$ 的面积，而 V 是四边形 $PQRS$ 的面积，求 $W = \dfrac{U}{V}$ 的值。



$W =$

Hong Kong Mathematics Olympiad (2018/19)
Finals (Group – Event 3)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+ Bonus score				Time		
			Total score					Min.	Sec.

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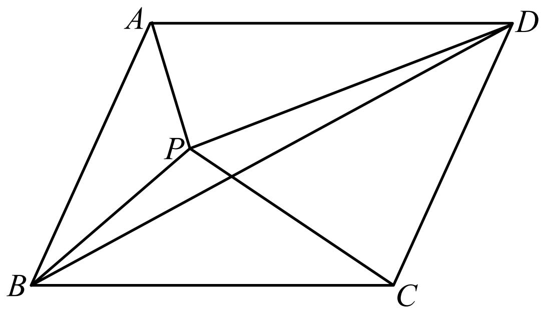
除非特别声明，答案须用数字表达，并化至最简。

1. If $\sqrt{32\times81\times343}=b\sqrt{a}$, where a and b are positive integers, determine the least value of a.
若 $\sqrt{32\times81\times343}=b\sqrt{a}$，其中 a 和 b 是正整数，求 a 的最小值。

$a=$

2. In the diagram below, point P is inside parallelogram $ABCD$. If the areas of ΔABP, ΔBPC and ΔBPD are 73 cm^2, 100 cm^2, and e cm^2 respectively, determine the value of e.
下图中，P 点在平行四边形 $ABCD$ 内。若 ΔABP , ΔBPC 和 ΔBPD 的面积分别为 73 cm^2 、 100 cm^2 和 e cm^2，求 e 的值。



$e=$

3. A 3×3 magic square is filled with a number in each square such that the sum of the

three numbers in each row, column and the two main diagonals are equal. The partially completed grid is shown below. Determine the value of c .
在以下的 3×3 幻方中，每行、列和两斜行（对角线）的和相等。如下图所示，部份数值已经填上。求 c 的值。

c	16	20
2		

$c =$

4. If $X = 2^{2018} + 3^{2018}$ and d is its unit digit, determine the value of d .

若 $X = 2^{2018} + 3^{2018}$ 及 d 是其个位数，求 d 的值。

$d =$

Hong Kong Mathematics Olympiad (2018/19)
Finals (Group – Event 4)

FOR OFFICIAL USE

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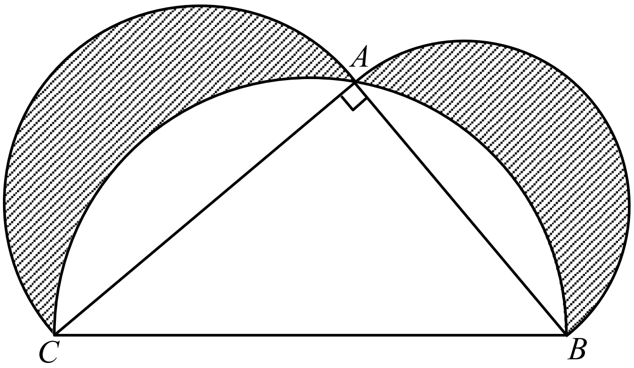
除非特别声明，答案须用数字表达，并化至最简。

1. ΔABC is a right-angled triangle with AC = 8 and BC = 10. Semi-circles are drawn with AB , AC and BC as diameters as shown. If the total shaded area is α , determine the value of α .
ΔABC 是直角三角形，$AC = 8$，$BC = 10$。以 AB、AC 和 BC 为直径分别画了三个半圆，如图所示。若阴影部分的总面积是 α，求 α 的值。



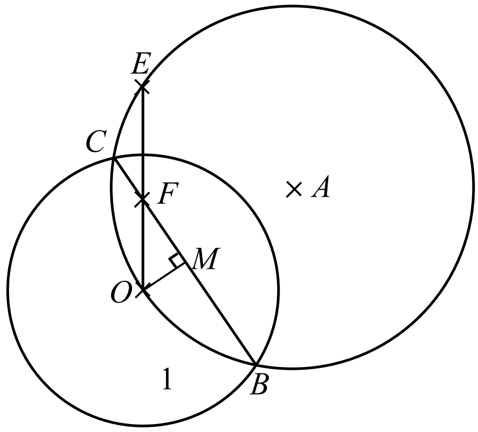
$\alpha =$

2. For all positive integers n, suppose there exists a function $F(n)$ defined for as follows:
$F(1)=0$,
for all $n \geq 2$,
$F(n)=F(n-1)+2$ if 2 divides n but 3 does not divide n;
$F(n)=F(n-1)+3$ if 3 divides n but 2 does not divide n;
$F(n)=F(n-1)+4$ if 2 and 3 both divide n ;
$F(n)=F(n-1)$ if neither 2 nor 3 divides n .
If $\beta=F(4000)$, determine the value of β.
对所有的正整数 n，设某一个函数 $F(n)$ 有如下定义：
$F(1)=0$，
当 $n \geq 2$，
如果 n 只能被2整除而不能被3整除，则 $F(n)=F(n-1)+2$；
如果 n 只能被3整除而不能被2整除，则 $F(n)=F(n-1)+3$；
如果 n 既能被2整除又能被3整除，则 $F(n)=F(n-1)+4$；
如果 n 既不能被2整除又不能被3整除，则 $F(n)=F(n-1)$。
若 $\beta=F(4000)$，求 β 的值。

$\beta =$

3. Two circles intersect at B , C as in the figure. If M is the mid-point of BC . $OM = 1$, $OC = 3$, $OE = 5$. If $\gamma = OF$, determine the value of γ .

如图所示，两圆相交于 B、C 两点。M 是 BC 的中点。$OM = 1$, $OC = 3$, $OE = 5$。若 $\gamma = OF$，求 γ 的值。



$\gamma =$

4. If $f(x) = \left(x + \frac{1}{2000}\right) \times \left(x + \frac{1}{2001}\right) \times \cdots \times \left(x + \frac{1}{2019}\right)$ and $\delta = f(1) - 1$, determine the value of δ.

若 $f(x) = \left(x + \frac{1}{2000}\right) \times \left(x + \frac{1}{2001}\right) \times \cdots \times \left(x + \frac{1}{2019}\right)$ 以及 $\delta = f(1) - 1$，求 δ 的值。

$\delta =$